Potentials between static SU(3) sources in the fat-center-vortices model

Sedigheh Deldar
Washington University, St. Louis, MO, 63130
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The potentials between static sources in various representations in $SU(3)$ are calculated based on the fat-center-vortices model of Faber, Greensite and Olejník. At intermediate distances, potentials are in qualitative agreement with "Casimir scaling," which says that the string tension is proportional to the quadratic operator of the representation. At large distances, screening occurs for zero-triality representations; for the representations with non-zero triality the string tension equals that of the fundamental representation.

I. INTRODUCTION

Besides numerical calculations, there have been attempts to introduce models which explain quark confinement. The center vortices model [1,2] introduced in the late 1970's by 't Hooft is one of those attempts. A center vortex is a topological line-like (in D=3 dimensions) or surface-like (in D=4 dimensions) field configuration which has some finite thickness. The vortex carries magnetic flux quantized in terms of elements of the center of the group. The fluxes form narrow tubes with constant energy per unit length (surface). In order for the vortex to have finite energy per unit length, the gauge potential at large transverse distances must be a pure gauge. However, the gauge transformation which produces that potential is non-trivial. It is discontinuous by an element of the gauge center. It is the non-trivial nature of the gauge transformation which forces the vortex core to have non-zero energy and makes the vortex topologically stable. Faber, Greensite and Olejník [3] introduced fat-center-vortices to obtain confinement of both fundamental and higher representation static sources. According to the fat-center-vortices model, the vacuum is a condensate of vortices of some finite thickness. Confinement is produced by the independent fluctuations of the vortices piercing each unit area of a Wilson loop.

Faber *et al.* explicitly worked out the model for SU(2). Here, I give the results of applying their model to SU(3). For completeness, I first briefly explain their model and then apply it to SU(3), using it to study the potentials between static quarks for the fundamental and a few other representations.

II. THE MODEL OF FABER, GREENSITE AND OLEJNÍK

In the fundamental representation of $SU(N)$, a center vortex linked to a Wilson loop, has the effect of multiplying the Wilson loop by the gauge group center,

$$W(C) \rightarrow \exp^{\frac{2\pi i n}{N}} W(C) \qquad n = 1, 2, ..., N-1. \tag{2.1}$$

Based on the vortex theory, the area law for a Wilson loop is due to the quantum fluctuations in the number of center vortices linking the loop. Adjoint Wilson loops are not affected by center vortices, unless the vortex core overlaps the perimeter of the loops. The fat-center-vortices model can explain confinement and the Casimir scaling of higher representation string tensions, if the vortex thickness is large enough. The average Wilson loop predicted by this model has the following form:

$$< W(C) >= \prod_x \{1 - \sum_{n=1}^{N-1} f_n (1 - Re\mathcal{G}_r[\vec{\alpha}^n_C(x)])\}, \tag{2.2}$$

$$\sigma_C = -\frac{1}{A} \sum_x ln\{1 - \sum_{n=1}^{N-1} f_n (1 - Re\mathcal{G}_r[\vec{\alpha}^n_C(x)])\}. \tag{2.3}$$

where A is the area of the loop C and $\mathcal{G}_r$ is defined as:

$$\mathcal{G}_r[\vec{\alpha}] = \frac{1}{d_r} Tr \exp[i\vec{\alpha}.\vec{H}]. \tag{2.4}$$

d_r is the dimension of representation r, and $\{H_i\}$ is the subset of the generators needed to generate all elements of the center of the group. (For $SU(3)$, λ_8 is sufficient.) Vortices of type n and $N-n$ are considered the same, except that the magnetic fluxes are pointed in opposite directions.

$$f_n = f_{N-n} \quad and \quad \mathcal{G}_r[\vec{\alpha}^n_C(x)] = \mathcal{G}^\star_r[\vec{\alpha}^{N-n}_C(x)]. \tag{2.5}$$

The parameter f represents the probability that any given unit area is "pierced" by a vortex; *i.e.*, a line running through the center of the vortex tube intersects the area.

The parameter $\alpha_C(x)$ depends on the vortex location; in other words, it depends on what fraction of the vortex core is enclosed by the Wilson loop. Therefore $\alpha_C(x)$ depends on both the shape of the loop and the position $\vec{x}$ of the center of the vortex (in the plane of the loop C) relative to the perimeter. For example, for SU(2) $\alpha_C(x)$ is equal to 2π, if the core is entirely inside the minimal area of the loop. It is zero if the core is entirely outside the minimal area of the loop. $\alpha_C(x)$ can be chosen to be [3]:

$$\alpha_R(x) = \pi[1 - \tanh(ay(x) + \frac{b}{R})], \tag{2.6}$$

in which a and b are constants, and

$$y(x) = \begin{cases} x - R & \text{for } |R-x| \le |x| \\ -x & |R-x| > |x| \end{cases}, \tag{2.7}$$

where R is the distance between two adjoint sources. x denotes the x-coordinate of the center of a vortex where it pierces the $x-t$ plane. If the time-like sides of the loop are at $x=0$ and $x=R$; then with α_R defined in Eq. 2.6:

1. For fixed R, as $x \to \pm\infty$, $\alpha_R(x) \to 0$.
2. $\alpha_R(x) = 2\pi$, if the vortex core is entirely contained within the loop.
3. As $R \to 0$, the percentage of any vortex core which is contained inside the loop goes to zero and $\alpha_R(x) \to 0$.

In Eq. 2.3, by expanding the logarithm to leading order in f_n, expanding $\mathcal{G}_r[\vec{\alpha}]$ to leading order in $\vec{\alpha}$, and using the identity

$$\frac{1}{d_r} Tr(H_i H_j) = \frac{C_2(r)}{N^2-1}\delta_{ij}, \tag{2.8}$$

one finds

$$\sigma_C = \frac{1}{A}\{\sum_x \sum_{n=1}^{N-1} \frac{f_n}{2(N^2-1)} \alpha^n_C(x).\alpha^n_C(x)\} C_2(r), \tag{2.9}$$

$C_2(r)$ is the eigenvalue of the quadratic Casimir operator of the $SU(N)$ group in representation r.

With the approximation of Eq. 2.9, the ratio of the string tension of representation r to 3(fundamental) must be proportional to the ratio of the eigenvalue of quadratic Casimir operator of representation r to 3(fundamental). This gives a "Casimir scaling regime."

Since the parameters $\alpha_C(x)$ depend on loop size, it is not trivial that σ_c is constant in the adjoint representation. Even if the adjoint potential were approximately linear in some interval, it is not obvious that the fundamental potential would be linear in the same range of distances. Applying their model to SU(2), Faber *et al.* were able to find a region in which the potentials for the fundamental, adjoint and $j=\frac{3}{2}$ representations are linear. For large distances, the adjoint potential was screened, and the $j=\frac{3}{2}$ representation potential changed its slope to be the same as fundamental one.

Even though the fat-center-vortices model predicts some of the expected behavior of the potential between static quarks, it has its own limitation, in particular, it violates the fact that the potential should be always a convex function of distance [4].

III. APPLYING THE FAT-CENTER-VORTICES MODEL TO $SU(3)$

Back to Eq. 2.3, $f_n = f_{N-n}$ as mentioned earlier. For the two types of the vortices in $SU(3)$, $f_1 = f_2 = f$ and $Re\mathcal{G}_r[\vec{\alpha}_C^1(x)] = Re\mathcal{G}_r[\vec{\alpha}_C^2(x)] = Re\mathcal{G}_r[\vec{\alpha}_C(x)]$. Therefore, the potential between static sources in representation r of $SU(3)$ is seen to be

$$V_r(R) = -\sum_{m=-\infty}^{+\infty} ln\{(1 - 2f(1 - Re\mathcal{G}_r[\vec{\alpha}_C(x_m)]))\}, \tag{3.1}$$

where $x_m = m + \frac{1}{2}$. I use Eq. 2.6 for α_c except the normalization factor is changed to $\frac{2\pi}{\sqrt{3}}$ for SU(3). The parameters a and b in Eq. 2.6 and f in Eq. 3.1 are free parameters of the model.

To find the potential $V_r(R)$, first I need to find H_i in Eq. 2.4 for each representation: 3, 6, 8, 10, 15-symmetric, 15-antisymmetric, and 27. For the fundamental representation, $H_1 = T_8 = \frac{\lambda_8}{2}$; where λ_8 is the diagonal Gell-Mann matrix.

To find T_8^r of other representations, by using the tensor method, I define $\{X_r^i; i = 1, ..., d_r\}$ which are the basis vectors for the space on which the representation act. The corresponding generators are obtained from [5]

$$[T_a^{D_1 \otimes D_2}]_{ix,jy} = [T_a^{D_1}]_{ij}\delta_{xy} + \delta_{ij}[T_a^{D_2}]_{xy}. \tag{3.2}$$

$T_a's$ are the group generators for representations D_1, D_2, $D_1 \otimes D_2$. The elements of T_8^r can be found by

$$T_8^r X_r^i = \sum_{j=1}^{d_r} C_{ij} X_r^j. \tag{3.3}$$

Fig. (1) shows the potentials for various representations versus R, in the range $R \in [1, 20]$. Parameters a and b in Eq. 2.6 are equal to .05 and 4 respectively, and f in Eq. 3.1 is equal to 0.1. It can be seen that, for each representation, there exists a region in which the potential is approximately linear. Fig. 2 plots the ratios of the potential of each representation to that of the fundamental representation. These ratios start out roughly at the ratios of the corresponding Casimir ratios which are 2.5, 2.25, 4.5, 7, 4 and 6 for representations 6, 8, 10, 15-symmetric, 15-antisymmetric and 27, respectively. So at least for some region, a rough agreement with Casimir scaling can be observed. The linear behavior of the potential and its proportionality to Casimir ratio at small and intermediate distances, have been found in numerical simulations [6,7] for various representations in SU(3). Fig. 3 plots the potentials for the range of $R \in [1, 100]$. Screening occurs for representations 8, 10 and 27 while the slope of the potentials for representations 6, 15-symmetric and 15-antisymmetric changes to the slope of the fundamental representation. Note the non-convexity near $R = 0$ for all representations, and especially in the range $R = 20$ to $R = 45$ for 15-symmetric and the range $R = 20$ to $R = 40$ for representation 15-antisymmetric. The non-convexity does not go away when another form of function α_c in Eq. 2.5 is used.

Screening can be understood as follows: Each representation can be labeled by the ordered pair (n, m), with n and m the original number of 3 and $\bar{3}$ which participated in constructing the representation. Triality is defined as (n-m) mod 3. Screening occurs for representations with zero triality: $8 \equiv (1, 1)$, $10 \equiv (3, 0)$, and $27 \equiv (2, 2)$. For these representations, as the distance between the two adjoint sources increases, the potential energy of the flux tube rises. A pair of gluons pops of vacuum when this energy is equal or greater than the twice of glue-lump mass. (A glue-lump is the ground state hadron with a gluon field around a static adjoint source.) For large distances, the static sources combine with the octet(8) charges (dynamic gluons) popped out of the vacuum and produce singlets which screen. Therefore the potential between static sources is no longer R dependent. Static sources in representations 10 and 27 transform into the 8(adjoint) first and then 8 transforms into the singlet by interacting with the gluonic field.

$$8 \otimes 8 = 27 \oplus \bar{10} \oplus 10 \oplus 8 \oplus 1, \tag{3.4}$$

$$10 \otimes 8 = 8 \oplus 10 \oplus 27 \oplus 35, \tag{3.5}$$

$$27 \otimes 8 = 64 \oplus 27 \oplus 27 \oplus 35 \oplus \bar{35} \oplus 10 \oplus \bar{10} \oplus 8. \tag{3.6}$$

Static sources in representations with non-zero triality, $6 \equiv (2, 0)$, $15_s \equiv (4, 0)$ and $15_a \equiv (2, 1)$, transform into the lowest order representation (3 and $\bar{3}$) by binding to the gluonic $8's$ which are popped out of the vacuum:

$$6 \otimes 8 = \bar{3} \oplus 6 \oplus 15 \oplus 24, \tag{3.7}$$

$$15_a \otimes 8 = 42 \oplus \bar{24} \oplus 15_a \oplus 15_a \oplus \bar{6} \oplus 3 \oplus 15_s, \tag{3.8}$$

$$15_s \otimes 8 = 48 \oplus 42 \oplus 15_s \oplus 15_a. \tag{3.9}$$

15-symmetric changes to 15-antisymmetric first, so it needs to interact with the 8's (popped from the vacuum) twice to transform to 3. Screening does not occur for representations with non-zero triality, since there is no way to get a zero triality representation by crossing a non-zero one with any number of 8's. As a result, the slope of the linear potentials of the representations with non-zero triality changes to the slope of the fundamental one, and a universal string tension is observed for large R. The representation 15-symmetric requires a larger value of R to approach the fundamental slope than representations 6 or 15-antisymmetric – presumably this is because two pairs of 8's must be popped from the vacuum in the 15-symmetric case.

IV. CONCLUSION

By applying the fat-center-vortices model to $SU(3)$, I showed that for each representation at intermediate distances, there exists a region in which the static potential is linear and qualitatively in agreemen inwith Casimir scaling. This is also in agreement with the observation in SU(3) simulations of a linear potential in proportion to Casimir ratio of the representation [6,7]. At large distances, zero-triality representations will be screened and the potential for non-zero triality representations parallel to the one for the fundamental representation.

V. ACKNOWLEDGEMENT

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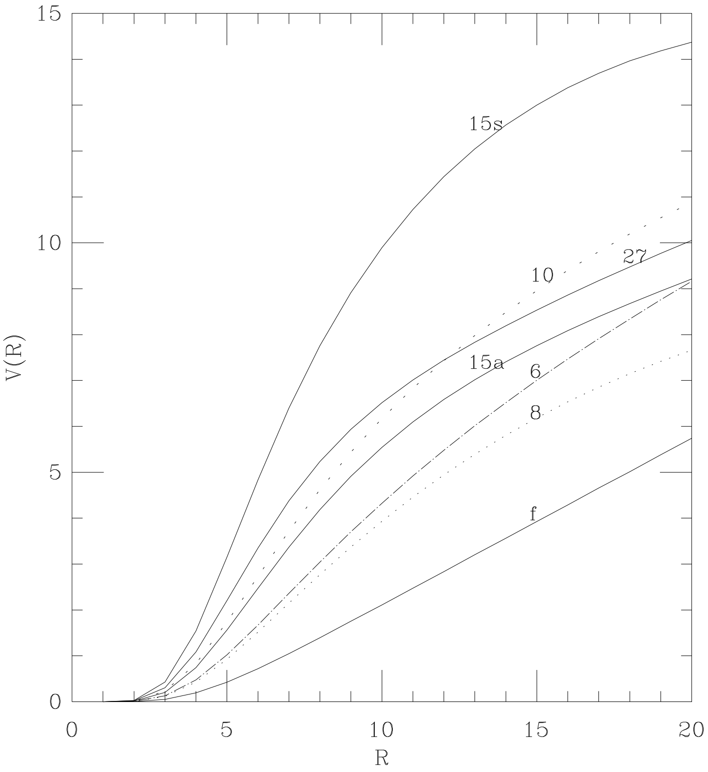

FIG. 1. Potential between static sources induced by fat-center-vortices model, for various representations. In the model, the scale of $V(R)$ and R are arbitrary (adjustable). The fundamental representation is shown by the letter “f”.

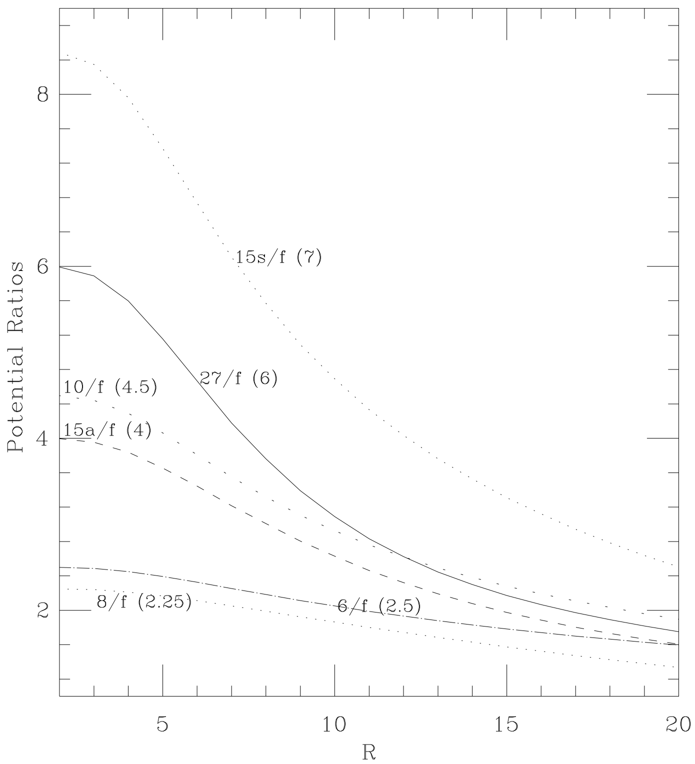

FIG. 2. Potential ratio of each representation to the fundamental representation (f). The ratios are qualitatively in agreement with the corresponding Casimir ratios (shown in parenthesis). The scales of R and $V(R)$ are arbitrary.

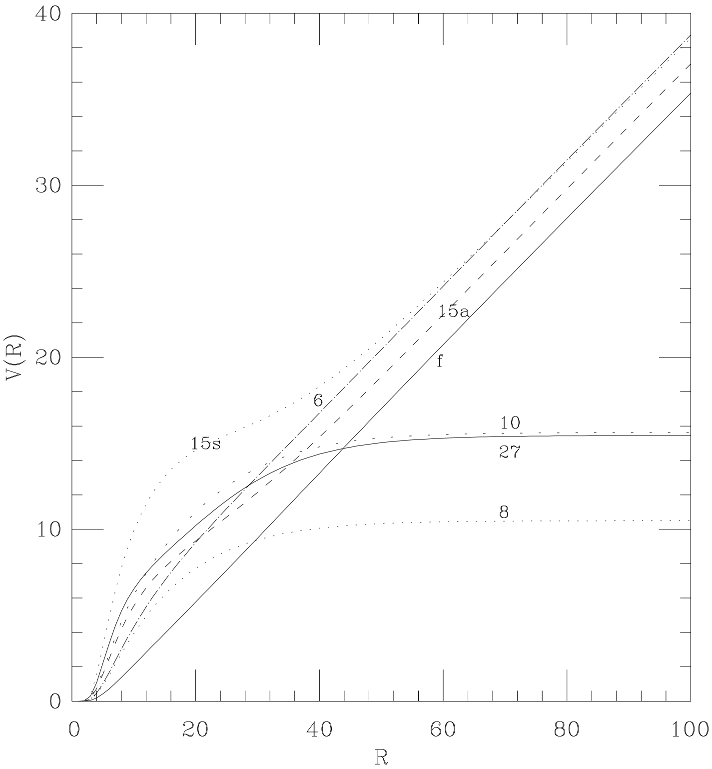

FIG. 3. Static sources potential for the range of $R \in [1, 100]$. For $R > 40$, potentials for representations with zero triality are screened and for the ones with non-zero triality closely parallel the fundamental potential. In the model, the scale of $V(R)$ and R are arbitrary (adjustable). The fundamental representation is shown by the letter "f".